Topic 10
Trigonometric Identities and Equations

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

PEARSON

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 26

Q1

(a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(2)

(b) Solve, for $0 \leqslant x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(4)

(Total for Question 1 is 6 marks)

Q2

Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(Total for Question 2 is 3 marks)

Q3

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} = 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 5\sin x$$

(3)

(Total for Question 3 is 7 marks)

Q4

Solve, for $0 \leq x < 360°$,

(a) $\sin(x - 20^\circ) = \frac{1}{\sqrt{2}}$,

(4)

(b) $\cos 3x = -\frac{1}{2}$.

(6)

(Total for Question 4 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$5\sin x = 1 + 2(1 - \sin^2 x)$	M1
	$2\sin^2 x + 5\sin x - 3 = 0$ (*)	A1cso (2)
(b)	$(2s-1)(s+3) = 0$ **giving** $s =$	M1
	$[\sin x = -3$ has no solution$]$ so $\sin x = \frac{1}{2}$	A1
	$\therefore\ x = 30,\ 150$	B1, B1ft (4) **[6]**
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$) A1 need 3 term quadratic printed in any order with =0 included	
(b)	M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or sinx) A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ or $x = \sin^{-1}\frac{1}{2}$ $y = \frac{1}{2}$ is A0 (unless followed by $x = 30$) B1 for 30 (α) not dependent on method 2nd B1 for 180 - α provided in required range (otherwise 540 - α) Extra solutions outside required range: Ignore Extra solutions inside required range: Lose final B1 Answers in radians: Lose final B1 S.C. Merely writes down two correct answers is M0A0B1B1 Or $\sin x = \frac{1}{2}$ $\therefore$ $x = 30,\ 150$ is **M1A1B1B1** Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1 NB Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$ $[\sin x = 3$ gives no solution$]$ $\sin x = -\frac{1}{2}$ $\Rightarrow$ $x = 210,\ 330$ This earns M1 A0 B0 B1ft Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30°, 150°$ This would be M1 A0 B1 B1	

Q2.

Question Number	Scheme		Marks
	Way 1	Way 2	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$	$2=(3\sin x-1)^2$ gives $9\sin^2 x-6\sin x+1=2$ so $\sin^2 x+8\sin^2 x-6\sin x+1=2$	B1
	E.g. $9\sin^2 x-6\sin x=1$ or $9\sin^2 x-6\sin x-1=0$ or $9\sin^2 x-6\sin x+1=2$	so $8\sin^2 x-6\sin x=1-\sin^2 x$	M1
	So $9\sin^2 x-6\sin x+1=2$ or $(3\sin x-1)^2-2=0$ so $(3\sin x-1)^2=2$ or $2=(3\sin x-1)^2$ *	$8\sin^2 x-6\sin x=\cos^2 x$ *	A1cso*
			(3)
			3

	Notes
	Way 1 B1: Uses $\cos^2 x = 1-\sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2=(3\sin x-1)^2$. If sin is used throughout instead of sinx it is A0. **Way 2** B1: **Needs correct expansion** and **split** M1: Collects $1-\sin^2 x$ together A1*: Conclusion and no errors seen

Q3.

Question	Scheme	Marks	AOs
(a)	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+5\sin x=4-5\cos x\Rightarrow\tan x=-1$	M1	2.1
	$x=135°, 215°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Q4.

Question Number	Scheme	Marks
(a)	45 (α)	B1
	$180-\alpha$, Add 20 (for at least one angle)	M1 M1
	65 155	A1 (4)
(b)	120 or 240 (β):	B1
	$360-\beta$, $360+\beta$	M1 M1
	Dividing by 3 (for at least one angle)	M1
	40 80 160 200 280 320	A1 A1 (6)
		(10 marks)

Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

(i) Solve, for $-180° \leq \theta < 180°$,

$$(1 + \tan\theta)(2\sin\theta - \sqrt{3}) = 0$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$2\sin x = \sqrt{2}\tan x.$$

(6)

(Total for Question 1 is 10 marks)

Q2

(a) Show that the equation

$$\tan 2x = 2\sin 2x$$

can be written in the form

$$(1 - 2\cos 2x)\sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leq x \leq 180°$,

$$\tan 2x = 2\sin 2x$$

You must show clearly how you obtained your answers.

(5)

(Total for Question 2 is 7 marks)

Q3

(a) Show that the equation

$$8\sin^2\theta - 2\cos^2\theta = 3$$

can be written as

$$10\sin^2\theta = 5.$$

(2)

(b) Hence solve, for $0° \leq \theta < 360°$, the equation

$$8\sin^2\theta - 2\cos^2\theta = 3,$$

(7)

(Total for Question 3 is 10 marks)

Q4

(a) Show that the equation

$$\sin\theta\tan\theta = \cos\theta + 1$$

can be written in the form

$$2\cos^2\theta + \cos\theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\sin\theta\tan\theta = \cos\theta + 1$$

showing each stage of your working.

(5)

(Total for Question 4 is 8 marks)

End of Questions

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
Q (i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
	$\sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 60, 120$	B1, B1ft (4)
(ii)	$2\sin x = \frac{\sqrt{2}\sin x}{\cos x}$	M1
	$2\sin x\cos x = \sqrt{2}\ \sin x \ \Rightarrow \quad \sin x(2\cos x - \sqrt{2}) = 0$	M1
	$x = 0, \ 180$ <u>seen</u>	B1, B1
	$x = 45, 135$	B1, B1ft (6)
		[10]

Q2.

Question number	Scheme		Marks
(a)	States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$		M1
	$\frac{\sin 2x}{\cos 2x} = 2\sin 2x \Rightarrow \sin 2x - 2\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1 - 2\cos 2x) = 0$ *		A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{2}$ gives $2x = 60$ **or** $2x = 300$		M1
	$x = 30, 150$		A1, A1 (5)
			7 marks

Q3.

Question Number	Scheme	Marks
(a)	$8\sin^2\theta - 2\cos^2\theta = 3$	
here	$8\sin^2\theta - 2(1 - \sin^2\theta) = 3$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$)	M1
	$8\sin^2\theta - 2 + 2\sin^2\theta = 3$	
	$10\sin^2\theta = 5$ cso AG	A1 (2)
(b)	$\sin^2\theta = \frac{1}{2}$, so $\sin\theta = (\pm)\frac{1}{\sqrt{2}}$	M1
	Attempt to solve both $\sin\theta$ = +.. and $\sin\theta$ = - ...(may be implied by later work)	M1
	$\theta = 45°$ (dependent on first M1 only)	A1
	θ $(= 180° - 45°)$; $= 135°$	M1; A1 √
	[f.t. dependent on first M and 3rd M]	
	$\sin\theta = -\frac{1}{\sqrt{2}}$	
	$\theta = 225°$ and $315°$	M1A1 (7) [9]

Q4.

Question Number	Scheme	Marks
(a)	$\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \cos\theta + 1$	M1
	$\left(\frac{1-\cos^2\theta}{\cos\theta}\right) = \cos\theta + 1$	dM1
	$1-\cos^2\theta = \cos^2\theta + \cos\theta \Rightarrow 0 = 2\cos^2\theta + \cos\theta - 1$	A1 cso (3)
(b)	$(\cos\theta + 1)(2\cos\theta - 1) = 0$	
	$\cos\theta = -1$ $\cos\theta = \frac{1}{2}$	M1
	One solution is $60°$ or $300°$, Two solutions are $60°$ and $300°$	A1, A1
	$\theta = \{60, 180, 300\}$	M1 A1 (5)

Gold Questions

Calculators may not be used

The total mark for this section is 32

Q1

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$90\sin(\theta + 60°) = 45$$

You must show each step of your working.

(4)

(ii) Solve, for $-180 \leq x < 180$, the equation

$$\tan x - \sqrt{2}\sin x = 0$$

(5)

(Total for Question 1 is 9 marks)

Q2

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \text{ ,, } k \text{ ,, } 3\,3\;\;3$$

3

(a) Find $\cos x$ in terms of k.

(3)

(Total for Question 2 is 6 marks)

Q3

Solve, for $0 \leq x < 180^{\circ}$,

$$\cos(3x - 10^{\circ}) = \frac{1}{\sqrt{2}}$$

You should show each step in your working.

(7)

(Total for Question 3 is 7 marks)

Q4

(i) Find the solutions of the equation $\sin(3x - 15^{\circ}) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180^{\circ}$

(6)

(ii)

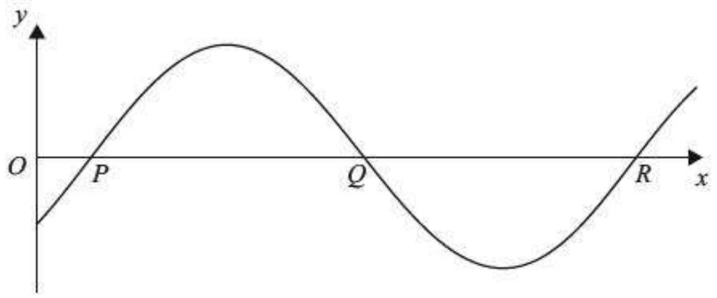


Figure 4

Figure 4 shows part of the curve with equation

$y = \sin(ax - b)$, where $a > 0$, $0 < b < 180$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are (11, 0), (108, 0) and 198, 0) respectively, find the values of a and b.

(4)

(Total for Question 4 is 10 marks)

End of Questions

<u>Gold Mark Scheme</u>

Q1.

Question Number	Scheme		Marks
	(i) $90\sin(\theta+60)=45$; $0\le\theta<360^\circ$ (ii) $\tan x-\sqrt{2}\sin x=0$; $-180\le x<180$		
(i)	$\sin(\theta+60^\circ)=\frac{1}{2}$ so $(\theta+60^\circ)=30^\circ$ $(\alpha=30^\circ)$	Sight of $\sin^{-1}\left(\frac{1}{2}\right)$	M1
	So, $\theta+60^\circ=\{150, 390\}$	$\theta+60^\circ$ = either "$180-$ their α" or "$360^\circ+$ their α" and not for θ = either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{90, 330\}$	A1: At least one of awrt 90° or awrt 330° A1: Both 90, and 330	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1for any extra solutions in range		
			[4]
(ii)	$\left(\frac{\sin x}{\cos x}\right)-\sqrt{2}\sin x=0$	Applies $\tan x=\frac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $\tan x-\sqrt{2}\sin x=0\Rightarrow\tan x(1-\sqrt{2}\cos x)$		
	$\sin x-\sqrt{2}\sin x\cos x=0$		
	$\sin x(2-3\cos x)=0$		
	$\cos x=\frac{1}{\sqrt{2}}$	$\cos x=\frac{1}{\sqrt{2}}$	A1
	$x=\{45, -45\}$	A1: One of either 45, or -45 A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le 1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x=0\Rightarrow\}\ x=0$ and -180	**Both** $x=0$ **and** -180 $\sin x=0$ **In this part of the solution, ignore extra solutions in range.**	B1
	Note solutions are: $x=\{-180, -45, 0, 45\}$ Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Q2.

Question Number	Scheme		Marks
(i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = 60°$	Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = 60°$	M1
	Adds 180 or 360 to previous value of angle		M1
	So $\theta =$ 20,80,140 (all three, no extra in range)		A1 (3)
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$		dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent		A1 (3)
			6

Q3.

Question Number	Scheme		Marks
	$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45$ (α)		B1
	$3x - 10 = \alpha \Rightarrow x = \frac{\alpha + 10}{3}$	Uses their α to find x. Allow $x = \frac{\alpha \pm 10}{3}$ **not** $\frac{\alpha}{3} \pm 10$	M1
	$x = \frac{55}{3}$		A1
	$(3x - 10 =)360 - \alpha$	$360 - \alpha$	M1
	$x = \frac{325}{3}$		A1
	$(3x - 10 =)360 + \alpha$	$360 + \alpha$	M1
	$x = \frac{415}{3}$		A1

Q4.

Question number	Scheme	Marks
(i)	$\sin(3x-15)=\frac{1}{2}$ so $3x-15=30$ (α) and $x=15$	M1 A1
	Need $3x-15=180-\alpha$ or $3x-15=540-\alpha$	M1
	Need $3x-15=180-\alpha$ and $3x-15=360+\alpha$ and $3x-15=540-\alpha$	M1
	$x=55$ or 175	A1
	$x=$ 55 , 135 , 175	A1
		(6)
(ii)	At least one of $(18a-b)=$ 0	M1
	$(108a-b)=$ 180	
	or $(198a-b)=$ 360	
	If two of **above equations** used eliminates a or b to find one or both of these or uses period property of curve to find a	M1
	or uses other valid method to find either a or b	
	Obtains $a=2$	A1
	Obtains $b=36$	A1
		(4)

Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

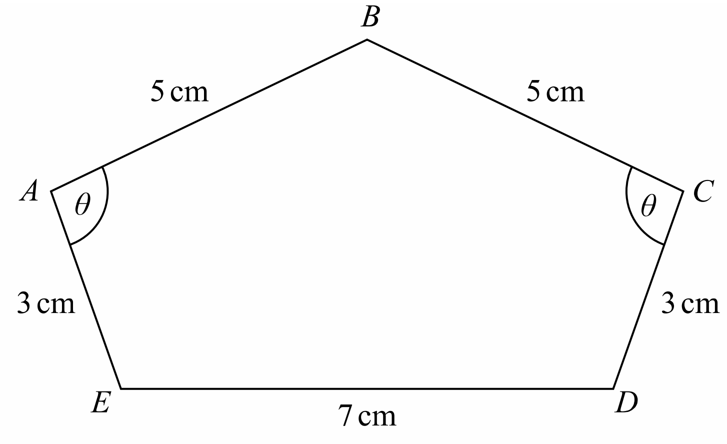


Figure 4

Figure 4 shows a shape $S(\theta)$ made up of five line segments AB, BC, CD, DE and EA.

The lengths of the sides are $AB = BC = 5$ cm, $CD = EA = 3$ cm and $DE = 7$ cm.

Angle BAE = angle $BCD = \theta$ radians.

The length of each line segment always remains the same but the value of θ can be varied so that different symmetrical shapes can be formed, with the added restriction that none of the line segments cross.

(a) Sketch $S(180^0)$, labelling the vertices clearly.

(2)

The shape $S(\phi)$ is a trapezium.

(b) Sketch $S(\phi)$ and calculate the value of ϕ.

(3)

The smallest possible value for θ is α, where $\alpha > 0$, and the largest possible value for θ is β, where $\beta > 180^0$.

(c) Show that $\alpha = \arccos\left(\frac{29}{40}\right)$. [arccos($x$) is an alternative notation for $\cos^{-1}(x)$]

(4)

(*d*) Find an expression for the value of β.

(4)

The area, in cm^2, of shape $S(\theta)$ is $R(\theta)$.

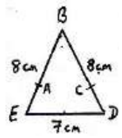
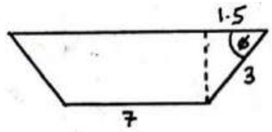
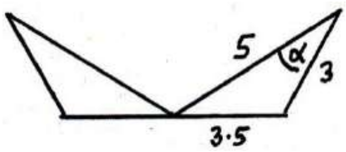
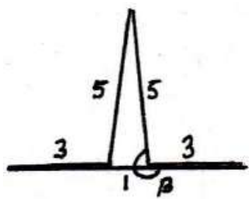
(*e*) Show that for $\alpha \leqslant \theta < 190^0$

$$R(\theta) = 15 \sin \theta + \frac{7}{4}\sqrt{87 - 120\cos\theta}$$

(4)

(Total for Question 1 is 17 marks)

Platinum Mark Scheme

Question	Scheme	Marks	Notes
7. (a)	Triangle EBD $EB = DB$ or labelling to show isos 	M1 A1 (2)	Needn't be isos D Correct labelling
(b)	Isos trapezium ($ACDE$) $\cos\phi = \dfrac{1.5}{3}$, so $\phi = 60^0$ 	B1 M1, A1 (3)	Sketch – with at least 1 side M1 for correct expression
(c)	$\cos\alpha = \dfrac{25+9-3.5^2}{2\times5\times3} = \left[\dfrac{29}{40}\right]$ (o.e.) So $\alpha = \arccos\left(\frac{29}{40}\right)$ **(*)** 	B1 B1 M1 A1cso (4)	Shape (o.e.) 2 or more side lengths Correct use of cos rule **In (c), (d) B1B1 can be implied by M1**
(d)	$\cos\left(\beta-180^0\right) = \dfrac{0.5}{5}$ $[=0.1]$ $\beta = 180^0 + \arccos\left(\frac{1}{10}\right)$ <u>or</u> $270^0 - \frac{1}{2}\arccos\left(\frac{49}{50}\right)$ 	B1 B1 M1 A1 (4)	Shape (o.e.) 2 or more side Correct expression (can ignore – p)
(e)	$BE^2 = 5^2+3^2-2\times5\times3\times\cos\theta = [34-30\cos\theta]$ [h = height from B to ED] so $h^2 = BE^2 - 3.5^2 = \left[\dfrac{87-120\cos\theta}{4}\right]$ Area = $15\sin\theta + \frac{1}{2}\times7\times\sqrt{\frac{87-120\cos\theta}{4}}$, $= \underline{15\sin\theta + \frac{7}{4}\sqrt{87-120\cos\theta}}$ **(*)**	M1 M1 M1,A1 (4)	Attempt BE or BD Attempt h M1 for correct areas A1 cso